On the Lunar Diurnal Variation of the Earth's Magnetism at Pavlovsk and Pola (1897-1903)

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IX. *On the Lunar Diurnal Variation of the Earth's Magnetism at Pavlovsk and Pola* (1897–1903).

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§ 1. The present paper contains an account of the methods and results of an investigation into the lunar diurnal magnetic variations at the observatories of Pavlovsk and Pola (1897–1903). This work is part of a larger undertaking, now in progress, by which I hope to obtain the data necessary for the discussion of the lunar diurnal magnetic variations over the whole earth, using the method of the Gaussian potential in a manner similar to that in which Schuster applied it to the solar diurnal magnetic variations. For this purpose data will be computed from at least three or four other observatories. The labour of computation involved is very large, and in the present instance has been executed partly by the aid of a grant from the Government Grant Committee of the Royal Society, and partly by assistance put at my disposal by the Astronomer Royal and Dr. Schuster. While the data thus obtained will, I trust, throw much new light on the theory both of the lunar and solar diurnal magnetic variations, the investigation has been undertaken not with any thought of finality in itself, but rather as an incentive to the provision of data for a more accurate and detailed discussion by subsequent workers. The reduction of the observations to the required form is, in fact, almost too great a labour for a single person to undertake if it is to be done on an adequate scale and in a reasonable time. It is in the hope, therefore, that a number of directors of magnetic observatories may be induced to carry out the reductions for their own stations that I have, in this paper, indicated what seems to me to be the simplest and most suitable method of computation for the purpose, and also some of the lines along which the discussion of the results from a single observatory should proceed. In a later paper some further points will be dealt with, in addition to the final discussion of the collected results from several observatories; for a general review of the subject, reference may be made to my recent discussion of the previously existing data for Batavia, Bombay,

and Trevandrum.*   A few of its main points are summarized below for the better understanding of the object of the present paper.

§ 2. The magnetic elements undergo regular variations with the period of a lunar day, just as they do in the course of a solar day ; the latter variations are much the larger of the two.   The lunar diurnal changes are the simpler in character, however, taking the form, when averaged over a whole lunation, of a purely semi-diurnal wave. But if the variation is computed from a number of days all at the same lunar phase (*i.e.*, when there is a definite angular distance between the sun and moon), the solar diurnal variation having first been eliminated from the observations, harmonic components of other frequencies in the lunar day are also found to be present.   The epoch of the component of frequency 2 (the semi-diurnal term) remains constant throughout the lunation, but the epochs of the terms of frequency 1, (2), 3, 4, ..., n change regularly during the lunation by amounts -2π, (0), 2π, 4π, ..., $2(n-2)\pi$ respectively.   Moreover, the epochs of the various components generally seem to be nearly or quite the same at the time of new moon, when the sun and moon are on the same meridian.   This law of phase change was discovered and verified by a study of the first four harmonic components ; it explains the disappearance of all components of frequency other than 2 from the variation computed from a whole lunation.   The phenomenon clearly suggests a solar action modifying a regular semi-diurnal lunar variation.   In the paper referred to, I have shown that the law according to which the epochs of the several harmonic components change during a lunation is in accordance with the theoretical consequences of such a solar action.   While the proof of this theorem might easily be stated in terms which imply no special hypothesis as to the manner in which the action takes place, it was actually given in terms of a particular theory, viz., that the effect is due to a variation, periodic in a solar day, of the electrical conductivity of the medium in which flow the currents to whose magnetic potential is attributed the variable field put in evidence by the observations.   The dependence of the electrical conductivity upon solar time is accounted for if we suppose that all or part of this conductivity is due to some ionizing influence from the sun.   So far as regards the facts thus mentioned, this assumption fits in naturally with a view that the lunar magnetic variations arise from the lunar tide in the earth's atmosphere† in the same way as, according to SCHUSTER's well-known theory, the solar diurnal magnetic changes arise from the ordinary daily atmospheric motions‡ which are indicated by the barometer.

In two important memoirs SCHUSTER has proved that the solar-diurnal magnetic

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* 'Phil. Trans.,' A, vol. 213, pp. 279–321.   This paper contains fairly full references to the previous work on the subject.

† This has been detected and measured in the barometric records of some Tropical observatories, *i.e.*, St. Helena and Batavia.   It is, as was to be expected, purely semi-diurnal.   SABINE, 'Phil. Trans., 1847 ; 'Batavian Observations,' 28, 1905.

‡ Mainly of solar-thermal rigin.

variations originate in currents flowing mainly above the earth's surface,* and has shown that these may very probably be ascribed to the effect of electromotive forces produced by the periodic motions of the atmosphere across the earth's permanent magnetic field, the electrical conductivity necessary for the passage of the currents being found in the rarified upper atmosphere, which is assumed to be ionized by some solar action such as ultraviolet radiation.† This theory is so valuable and successful in many ways that it is desirable to examine carefully the points not yet elucidated by it, among which the chief are the ratios of the amplitudes of the several harmonic components in the magnetic variation (as compared with the ratios of the corresponding terms in the barometric variation), and the seasonal changes in these components. If the natural assumption that the mechanism of the lunar and solar diurnal magnetic variations is similar is correct, an investigation of the former should throw much light on the latter; since the lunar diurnal variation arises from an atmospheric oscillation of a single frequency there is good reason to suppose that the relative amplitudes of the various components of the lunar magnetic variation (computed for the separate lunar phases and reduced to the time of new moon, say) will enable the daily variation of the atmospheric electrical conductivity to be determined.

The chief point of novelty and importance in the investigation for which the data here presented will be employed lies in the use of the harmonic components of the lunar magnetic variation of frequencies other than 2, as well as the main semi-diurnal component. In particular, as is explained in the memoir already referred to,‡ the fourth harmonic term has a special significance for our purpose.

The Method of Computation.

§ 3. The following is the method of computing the lunar diurnal magnetic variation from the published hourly values of the magnetic elements which, after much consideration, has appeared to me to be the simplest and best. It resembles that used by J. A. Broun in his reduction of the Trevandrum observations of declination,§ more than any other method hitherto used.

The monthly mean solar diurnal variation, as ordinarily calculated from a calendar month's hourly observations, may be considered free from any lunar term, since in this period‖ the lunar disturbance characteristic of any lunar hour affects in turn each

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* Schuster, 'Phil. Trans.,' A, vol. 180, p. 467.

† Schuster, 'Phil. Trans.,' A, vol. 208, p. 163.

‡ 'Phil. Trans.,' A, vol. 213, p. 306.

§ 'Trevandrum Magnetical Observations,' vol. I., p. 113, 1874.

‖ More strictly, in an exact lunation, but the two are so nearly equal that in view of the relative smallness of the lunar variation the difference may be neglected.

group of 28 to 31 equal solar hours in like degree. If, then, we subtract from each hourly value the monthly mean value for that hour, the remainders will be due to causes not regularly solar, *i.e.*, to irregular disturbances from whatever cause, and to any lunar action which may be at work. These remainders, when re-grouped, in sufficient quantity, according to their lunar time, enable the magnetic variations depending on lunar time to be determined either for whole lunations, or for any particular lunar phase, since the regular solar variations are already eliminated, and the accidental irregularities, independent of lunar time, average out.*

In eliminating the solar variation it is too troublesome to subtract from the hourly values on each day the monthly mean values for the twenty-nine days exactly centred at that day, yet, on the other hand, it is perhaps hardly accurate enough to assume that the monthly mean values for the ordinary calendar month properly represent the solar variation for all the days in that month—especially during seasons of rapid change in the solar variation. The simple plan which I have adopted is to take the days of a calendar month in six groups each containing five days (except that the last group contains the days from the 26th to the end of the month), and to take the same monthly mean hourly values for all the days in each group; these hourly values are interpolated according to simple rules (embodied in a table for the computer's convenience) from the monthly mean hourly values of the given calendar month and of those preceding and following it, each set being assumed to apply to the centre of its respective month. The differences "actual—mean" are written down on the sheets containing the printed hourly values, alongside the latter; it is convenient, where possible, to place positive differences to the left, and negative differences to the right of the printed values. These differences need not be written down more accurately than to one-tenth of a minute of arc in the case of declination, or to 1γ (0.00001 C.G.S. unit) in the case of force.

The re-grouping of these differences according to their lunar time is effected by copying them on to "lunar" sheets ruled into 27 columns and 40 or 45 rows, on which have been written out, in the second column, the local civil times of upper transit of the moon (lunar hour 0) for the successive days in the calendar month to which each sheet is devoted. The times of lunar hour 0 will not generally fall at an exact solar hour, and the nearest solar hour to the correct time must be used instead; the resulting error will generally be small, and is assumed to disappear in the mean from a large number of days. These times, for the meridian of Greenwich, can be taken from the Nautical Almanac or, perhaps, more conveniently, from the "*Companion to The Observatory*" (12^h must be added to obtain *civil* instead of astronomical time). They may be corrected for difference of longitude for an observatory on another meridian, but I have found it convenient not to do so, using the same civil times of

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* It may be noted here that in the computation of the lunar magnetic variation many small errors of solar diurnal periodicity which occasionally occur in magnetic observations, such as those arising from a faulty temperature correction, are eliminated from the observations with the regular solar variation.

lunar hour 0 for all observatories; the necessary correction for this is indicated later.*

The length of a lunar day is not quite constant, but its mean value is approximately 24^h 50^m in solar time; for the present purpose it will be assumed to equal 25^h of solar time (which seems much preferable to the adoption of 24^h, as has often been done†). As the hourly differences to be copied out are given at intervals of one solar hour, the lunar day will, for the first stage of the work, be regarded as subdivided into 25 equal periods of duration of one solar hour, and the last 25 columns of the "lunar" sheets should be headed 0^h to 24^h. In copying out the hourly differences, therefore, the first value written down on any row (each of which relates to a single lunar day) is that one which corresponds to the local civil time written in the second column of the row; after this the next 24 consecutive hourly differences are written in order. The times of lunar hour 0 in the second column generally differ by successive amounts of 25^h, and where this is so the hourly differences run on consecutively from one row to the next; occasionally the difference is only 24^h, however, and in this case one hourly value has to be set down twice, once under column 24^h of the former row, and again under 0^h of the following row. To ensure that the right set of hourly differences is being copied out, the time of the last entry in each row of the lunar sheets should be checked. In this re-arrangement it is important, for convenience and accuracy, that the columns of the lunar sheets should be wide enough to allow of the preservation of the separation of the positive and negative values of the hourly differences, the former on the left and the latter on the right hand of the columns; this facilitates the formation of monthly and group sums, and obviates the use of signs. Each lunar sheet should contain the lunar days of a single calendar month, occupying about 30 rows; a few hours at the beginning or end of the month may be taken from the preceding or following calendar month in order to make each lunar day complete.

§ 4. When all the hourly differences for the period under investigation have been copied out on to the lunar sheets these should be carefully scanned for the purpose of picking out any very disturbed days for rejection. This assists also in the detection of any mistakes which have been made in the previous part of the work, as serious mistakes in forming the hourly differences in copying is generally apparent from the "run" of the numbers. As regards the rejection of disturbed days, I have adopted a very conservative policy; the proceeding must, in any case, be of an arbitrary nature, and a criterion which will mark off a given proportion of days must vary in different years, at different observatories, and with different elements (even at the same time and place). The period covered by this investigation (1897–1903) was intentionally chosen during a sunspot minimum, when quiet magnetic conditions prevailed, so that in a more disturbed period it would be necessary to reject a rather larger proportion

* P. 302. It is assumed that the hourly values are tabulated, as is usual, according to the local civil time of the observatory.

† As, for instance, at Batavia, Bombay, and in all SABINE's work on the lunar magnetic variation,

of days. The average total number of days rejected for each element at each observatory, out of nearly 2500 lunar days (7 years) was 50 or 60*, or about 2 per cent. of the whole. The rejected days were not always the same for different elements and observatories. At first a still smaller number, hardly more than a dozen days were deleted; the further rejection of another 40 days improved the accordance of the results from the several lunar phases, and was therefore considered to be worth while, although it hardly altered the final means from all the phases.

The number of days thus rejected is very much smaller than was customary amongst those previous investigators who adopted SABINE's method of "separating values"; CHAMBERS,† for example, in treating 25 years' Bombay observations, rejected about 20 per cent. of all the days, instead of 2 per cent. as here. FIGEE,‡ at Batavia, has stated that this procedure may seriously affect the values derived for the lunar magnetic variation, and it seems desirable, therefore, for definiteness and uniformity, to restrict the number of rejected days within very close limits.

The disturbed days, chosen according to the principles thus laid down, were ruled out on the lunar sheets, as also any lunar days with incomplete record. The monthly sums were then formed at the foot of the several columns of hourly differences—the corresponding number of days used being written in the second column of the same row. Further, by reference to the Nautical Almanac, each month was divided up into eight groups of days (there being from three to five days in each group), these being distributed as evenly as possible around the times§ of new moon, one-eighth phase, first quarter, and so on, as centres. In eight rows, below the monthly sums, should be written down the sums of the hourly differences for each of these groups of days in order (new moon being phase 1, first-eighth phase 2, and so on), the number of days in each group being set down in the second column. The sum of the eight group sums in any column should equal the monthly sum in that column, and the additions should be checked by verifying that this is the case.

§ 5. At this stage the utmost freedom is possible in the choice of further grouping of the results for discussion. In general, several months must be grouped together in order to obtain an accurate determination of the variation under any given conditions, as its total amount is always very small. The groups of days taken together should therefore be as large as is consistent with the exhibition of those features of the variation which it is desired to bring out. For the present purpose the eight lunar

* As an indication of the sort of criterion which serves to separate about 60 days for rejection, in the three elements declination, horizontal force, and vertical force at Pola, the separating values were respectively $10' \cdot 0$ of arc, 100γ, and 50γ in the total range of the hourly differences on any day. These limits might be different at other places and times.

† CHAMBERS, 'Phil. Trans.,' A, vol. 178, pp. 1–48 (1887).

‡ FIGEE, 'Batavian Observations,' vol. 26 (1903).

§ If the Greenwich civil times of lunar hour 0 are written in the second column of the lunar sheets, instead of the local civil times, the Greenwich civil times of the lunar phases should also be used in dividing up the months. This plan is recommended.

phases must, of course, be kept separate, and to illustrate the seasonal changes I have divided the twelve months of the year into three groups, summer (May to August), the equinoxes (March, April, September, October) and winter (November to February)—a plan which seemed better, on the whole, than the adoption of four groups of three months each.

During the seven years there were 28 group sums for each lunar phase during each season, and these were added together, the final sum representing about 100 days in each case (approximately 7 (years) × 4 (months) × 4 (days)).   The monthly sums were also added together for the seven years, so as to show the monthly change in the semi-diurnal component of the variation ; by grouping these monthly sums in seasons a check on the sum of the eight seasonal sums for the separate phases was obtained for verification of the computing.

The twenty-four sets of inequality sums for the eight lunar phases during three seasons were then plotted, and 24 ordinates at equal intervals were read off from each of these curves (without smoothing), in order to replace the 25 values at intervals of one solar hour by 24 values, starting at lunar hour 0, at intervals of a lunar hour, or $\frac{25}{24}$ solar hours ; this was done because the Fourier analysis of 24 values of a periodic function is arithmetically much simpler than that of 25.   This analysis, as far as the first four harmonic components, was then effected.   Only at this stage were means, and not sums, used, the resulting values of a_1, b_1, &c., being divided by the number of days corresponding to the sums whose succession of values was harmonically analysed.   This plan, besides being very economical of time, permits of more convenient checks being applied to the routine computations (as already mentioned) than would be possible if means were used in an earlier stage of the work.

The eight values of a, b in the formula

$$\sum_{n=1}^{4} (a_n \cos nt + b_n \sin nt)$$

deduced from the magnetic variation for each lunar phase were then all reduced to the corresponding values at new moon, by allowing for the progressive change of epoch of amount $2(n-2)\pi$ which the nth harmonic undergoes during each lunation.[*] The change of epoch in the interval (one-eighth of a lunation) between one phase and the next is $\frac{1}{4}(n-2)\pi$, or $-45°$, 0, $+45°$, $+90°$ for the first four harmonics in order. We will denote by a', b' the new values of a, b after this change of epoch has been allowed for by reducing a, b, to their equivalent values at new moon.   Writing k for $\frac{1}{\sqrt{2}}$, for convenience, the table on p. 302 is useful for the conversion of a, b to a', b'.

Except for accidental error, and a small effect due to the varying distance and declination of the moon, the values of a' or b' in any one column should be the same. Their mean gives the resultant determination of the various components of the lunar

* 'Phil. Trans.,' A, vol. 213, p. 287.

Lunar phase.	a'_1.	b'_1.	a'_2.	b'_2.	a'_3.	b'_3.	a'_4.	b'_4.
1	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4
2	$k(a_1+b_1)$	$k(-a_1+b_1)$	a_2	b_2	$k(a_3-b_3)$	$k(a_3+b_3)$	$-b_4$	a_4
3	b_1	$-a_1$	a_2	b_2	$-b_3$	a_3	$-a_4$	$-b_4$
4	$k(-a_1+b_1)$	$k(-a_1-b_1)$	a_2	b_2	$k(-a_3-b_3)$	$k(a_3-b_3)$	b_4	$-a_4$
5	$-a_1$	$-b_1$	a_2	b_2	$-a_3$	$-b_3$	a_4	b_4
6	$k(-a_1-b_1)$	$k(a_1-b_1)$	a_2	b_2	$k(-a_3+b_3)$	$k(-a_3-b_3)$	$-b_4$	a_4
7	$-b_1$	a_1	a_2	b_2	b_3	$-a_3$	$-a_4$	$-b_4$
8	$k(a_1-b_1)$	$k(a_1+b_1)$	a_2	b_2	$k(a_3+b_3)$	$k(-a_3+b_3)$	b_4	$-a_4$

variation for the particular season, over the period for which data have been used, except for small corrections to be applied (i) to the amplitudes of the components, on account of the fact that the epochs are changing continuously, while we have treated the group of days corresponding to each lunar phase as though each of the days of the group was centred precisely at that phase (this makes the computed amplitude smaller than it should be); and (ii) to the epochs also, in case the civil times adopted for lunar hour 0 and the phases of new moon, &c., have been the times appropriate to the meridian of Greenwich instead of to the meridian of the observatory dealt with.

The former correction, to the amplitudes, is effected by multiplying the computed amplitudes C_1, C_3, C_4, or the corresponding a, b coefficients, by $1\cdot02$, $1\cdot02$, and $1\cdot11$ respectively.*

§ 6. The correction to the epochs is found as follows: we consider a place in West longitude $L°$, for which the civil times of lunar hour 0 and of new moon first-eighth phases, &c., appropriate to Greenwich have been used instead of the *local* times. The error in the local civil time of lunar hour 0 is equal to the gain in longitude of the moon upon the sun during the time taken for the moon to traverse the angle L; this time is, with accuracy sufficient for our purpose, equal to $L°/29$ in lunar time. On this account an angle $nL°/29$ must be subtracted from the deduced epoch θ_n. Next as regards the hour of new moon, &c., since these phenomena have no relation to any special meridian, the adopted times are in error by the full amount $L°$ in solar time. This results in an error of amount $(n-2)L°/29$ in the reduction of the phases to the true epoch of new moon. Combining the two corrections it appears that the epochs of all the components must be corrected by the same amount, viz., $-2L°/29$. This correction is so simple to apply, and the adoption of one set of times for all observatories is so convenient, that the plan can be recommended, certainly for all observatories for which no direct ephemeris is published.

Having applied these corrections, it is desirable that the results from all observatories should be given in the following form: the elements should be transformed into the geographical components of force, to North, West, and the nadir, the unit being conveniently taken as 10^{-7} C.G.S.; the time zero should be the local

* *Cf.* 'Phil. Trans.,' A, vol. 213, p. 309, for the deduction of these factors.

time of upper culmination of the moon at the epoch of new moon, and the a, b and C, θ coefficients should both be given.

The Lunar Diurnal Magnetic Variations at Pavlovsk and Pola.

§ 7. In undertaking the present investigation it was necessary to limit, as far as possible, the quantity of observations dealt with, and this made it desirable to choose a period of years which were magnetically " quiet," in order to minimise the irregular deviations of the magnetic needle which have to be averaged out in determining the lunar diurnal variations. The period adopted was that from 1897 to 1903 inclusive, comprising seven years during the sunspot minimum immediately preceding the present one.

The choice of observatories to be first dealt with fell on Pavlovsk and Pola; both of these publish the hourly values of the magnetic elements, and their latitudes rendered them suitable for the main purpose of this work. The prior examination of these rather than of other observations was due to the greater novelty of results relating to the lunar magnetic variation in fairly high latitudes; the phenomenon has been studied in detail only at tropical observatories hitherto.

The following particulars concerning Pavlovsk and Pola are important for our purposes :—

	Pavlovsk.	Pola.
Latitude	59° 41′ 13″ N.	44° 51′ 49″ N.
Longitude	30 29 15 E.	13 50 46 E.
Declination (1900) . . .	0° 37′ E.	9° 25′ W.
Horizontal Force (1900) .	16550γ.	22210γ.

The correction to the epochs of the harmonic components of the variations, described in § 6, is therefore $+2°{\cdot}1$ in the case of Pavlovsk, and $+1°{\cdot}0$ in the case of Pola.

§ 8. The results from these two observatories are exhibited in Tables I. to VI. Tables IA. and IB. give a summary of the semi-diurnal components of the lunar diurnal magnetic variation for the three elements and three seasons (as well as for the mean of the whole year), in the form of hourly inequalities. These are obtained from the mean of a number of whole lunations, so that any component of period other than half a lunar day is to be regarded as merely accidental. As may be readily seen, the results exhibit very clearly in most cases the true semi-diurnal character of the constant term in the lunar variation.

The inequalities are given for twenty-five times, separated by intervals of one solar hour, there being twenty-five solar hours in a lunar day. The small correction to the epoch, mentioned in § 6, has not been applied, as this table is given only to illustrate the main feature of the lunar diurnal variation, i.e., its semi-diurnal character. It will be noticed that the lunar variation is inverted during the winter months in the case of declination, and partially so in the case of horizontal force.

§ 9. The next six tables, II. to IV. (A and B) are more important, being derived by the separation of the lunar phases, enabling us, as already explained, to determine the other harmonic components, of changing epoch, in the lunar variation. The values of the coefficients a_n and b_n in the formula $\Sigma (a_n \cos nt + b_n \sin nt)$, for the first four components, are given after reduction to the same lunar phase (new moon). No corrections to amplitude or epoch (§§ 5, 6) have been made at this stage, nor are the results transformed into the geographical components of force, as the purpose of these tables is to show how far the results from the different lunar phases are in agreement. Apart from accidental error any eight corresponding values of a or b should be equal (except for the small effect of the varying distance and declination of the sun and moon). Judged by ordinary standards the agreement of the numbers in any one column is in general not at all good, even in the case of the larger amplitudes; but for work of this kind, where the effect sought after is so small in comparison with the superposed regular and irregular variations which have to be eliminated in its determination, the results may be regarded as satisfactory considering (i) the extreme smallness of the unit, and (ii) the short period (on the average equal to 100 days) from which each of the values in Tables II. to V. is determined. Occasionally there are largely outstanding values, and it would seem necessary to use a much larger series of observations if close accordance of the values for the separate lunar phases were desired. I hope, however, that the mean coefficients deduced from the combination of the eight phases are reasonably free from accidental error—the parallelism of the results from the two observatories, which will be pointed out later, confirms this to some extent.

§ 10. In order, however, to examine this point more closely, I have derived the values of the probable errors of the mean values of a_n and b_n in each case from the discordances from the means in Tables II. to IV.* The results are given in Table V., those for declination (Tables IIIA. and IIIB.) having for convenience of comparison been converted into force units. The essential features of the table may be summarized thus : the sum of all the probable errors for all the elements in summer, equinox, and winter are 293,276 and 333 respectively, so that the season has no great effect upon the magnitude of the probable error ; but as the variations are generally greatest in summer, the values of a, b for this season are in general proportionately the most accurate. Summing the errors for all seasons and components, but keeping the results for each element and observatory separately, we find the following :—

	Horizontal force.	Transverse force.	Vertical force.
Pola	171	150	108
Pavlovsk	163	191	119

* The probable error of the mean of n observations is taken to equal $0 \cdot 845 / \sqrt{n-1}$ times the mean discordance from the mean.

Hence the probable errors of the results for vertical force are distinctly less than those for the other two elements, but here again, an account of the much smaller magnitude of the variations in vertical force, these are *proportionately* the least accurately determined.* These differences between the probable errors are, however, small compared with those between the errors relating to the different harmonic components. The means of the successive columns of Table V. (after the application of the small amplitude corrections of § 5) are

$a_1.$	$b_1.$	$a_2.$	$b_2.$	$a_3.$	$b_3.$	$a_4.$	$b_4.$
11·7	9·9	6·7	6·6	4·4	4·3	4·2	3·4

Hence the probable errors of either resultant of the harmonic components of frequency 1, 2, 3, 4, are respectively 10·8, 6·6, 4·3 and 3·8, in units of 10^{-7} C.G.S. These results apply directly to Table VIB.

§ 11. In Tables VIA. and VIB. are given the results of this paper in their complete form, the necessary corrections to amplitude and epoch having been applied to the mean values of a_n and b_n in Tables II. to IV., after which these were converted into the geographical components of force, with a common unit 10^{-7} C.G.S. The C, θ and a, b results are given separately in Tables VIA. and VIB. The latter is the more useful for the ultimate purpose of this work, viz., for the investigation of the potential of the lunar diurnal variations; the C, θ table, however, gives a better idea of the relations between the various components at a single observatory, and between corresponding components at the two observatories.

* The following comparison with the values deduced for the a_2, b_2 coefficients of the lunar diurnal variations at Pavlovsk and Pola by VAN BEMMELEN (' Meteorologische Zeitschrift,' May, 1912) may also be of interest. VAN BEMMELEN divided his data into the summer and winter half-years, and therefore the only comparison that can be made is between the mean variations throughout the whole year. Moreover, as he did not separate the different lunar phases, but took the mean of a number of whole lunations, the values of a_1, b_1 which he gives represent accidental error and are no true lunar effect (as he fully recognized); the real diurnal lunar component simply disappears from the result when whole lunations are dealt with. Further, the method of computation used by him was different, as also the period of observations dealt with (1894–1903 in the case of Pavlovsk, and 1899–1906 in the case of Pola). While exact agreement is not to be looked for, therefore, the following comparison has some interest in the present connection :—

Element.	Pavlovsk.				Pola.			
	$a_2.$		$b_2.$		$a_2.$		$b_2.$	
	VAN BEMMELEN.	CHAPMAN.	VAN BEMMELEN.	CHAPMAN.	VAN BEMMELEN.	CHAPMAN.	VAN BEMMELEN.	CHAPMAN.
North force . . .	11	0	15	48	52	42	59	56
West force	53	61	− 2	7	74	61	9	30
Vertical force . . .	10	0	12	0	−20	−27	37	27

First considering the phase angles, a general tendency to equality is evident between the values of θ for the four components of a single element and season; this result is obscured to some extent by irregularities, which are probably mainly accidental; it is what would be expected on the grounds of general theory (*cf.* § 23 of my former paper already cited). A comparison of corresponding phase angles at the two observatories reveals considerable similarity, though by no means exact agreement between the two sets of values. Both exhibit the more or less partial inversion, during the winter, of the variations in the horizontal force components, which does not appear in the case of vertical force (the vertical force variations for Pavlovsk are so small in amplitude that but little reliance can be placed upon the phase angles).

As regards the amplitudes, the second harmonic is generally the greatest, though the first harmonic (which is usually the next greatest) sometimes equals or even exceeds it. The fourth harmonic is generally the smallest of all, often being quite inappreciable. The first three harmonics are as a rule much greater in summer than in the other seasons, the winter values usually being the smallest. A remarkable feature, however, brought out by Table VIA. is that the fourth harmonic is always greatest at the equinoxes, both absolutely and (*à fortiori*) in proportion to the other components; this is shown even in the case of Pavlovsk vertical force. The equinoctial amplitude is frequently one-third that of the second harmonic, and sometimes greater. As I have previously shown (§ 25, p. 306, *loc. cit.*), the fact that the fourth harmonic is appreciable at all indicates that the diurnal variation in the atmospheric electrical conductivity cannot be represented by a simple diurnal wave $a + b \cos t$, but that terms of higher frequency in the solar day must be present. The occurrence of these large values of C_4 at the equinoxes should therefore have an interesting physical bearing; as yet, however, I have not entered into these theoretical questions, pending the acquisition of data from further observatories, and also the completion of an extension, by improved methods, of the mathematical theory of my former paper, both of which are now in hand.

TABLE IA.—Lunar Diurnal Magnetic Variations at Pola Determined from the Mean of a Number of Whole Lunations.*

Twenty-five hourly values are given, separated by intervals of one solar hour, there being twenty-five solar hours in a lunar day.

The unit is 10^{-7} C.G.S. for force, and $0'\cdot001$ for declination; + denotes force increasing to North, West, and the Nadir.

Lunar hour	Horizontal force				Declination				Vertical force			
	Summer	Equinox	Winter	Year	Summer	Equinox	Winter	Year	Summer	Equinox	Winter	Year
0	+97	+21	+28	+49	+278	+67	−80	+88	−62	−24	−36	−41
1	+109	+63	−10	+54	+221	+101	−18	+105	−32	−22	−17	−24
2	+123	+52	−31	+48	+169	+54	+27	+83	+8	+18	+4	+10
3	+74	+80	−46	+36	+58	−17	+70	+37	+56	+28	+23	+36
4	+58	+68	−41	+28	−69	−0	+45	−8	+74	+11	+30	+38
5	−20	+32	−41	−10	−179	−32	+73	−46	+79	+6	+34	+40
6	−89	−15	−30	−45	−257	−66	+61	−87	+63	−4	+13	+24
7	−154	−43	−23	−73	−201	−96	+18	−93	+39	−0	+3	+14
8	−188	−78	−9	−92	−165	−75	−40	−93	+14	−8	+9	+5
9	−183	−55	−32	−90	−75	−59	−42	−59	−12	−11	−6	−10
10	−72	−27	+0	−33	+28	+65	−50	+14	−45	−13	−37	−32
11	−22	+25	+13	+5	+171	+75	−77	+56	−45	+3	−30	−24
12	+66	+60	+25	+50	+237	+78	−108	+69	−38	+7	−22	−18
13	+130	+86	+15	+77	+212	+38	−21	+76	−27	+8	−26	−15
14	+156	+85	+22	+88	+152	+75	+26	+84	+13	+17	−6	+8
15	+169	+105	+24	+99	+46	+57	+55	+53	+22	+27	+2	+17
16	+122	+44	+15	+60	−65	+21	+63	+6	+45	+38	+19	+34
17	+63	+10	+19	+31	−135	−32	+87	−27	+50	+33	+35	+39
18	+6	−16	−8	−6	−190	−42	+83	−50	+39	+21	+46	+35
19	−62	−58	−6	−42	−235	−44	+11	−89	+34	+8	+41	+28
20	−112	−121	−9	−81	−198	−88	+21	−88	+10	−9	+17	+6
21	−110	−129	+27	−71	−142	−60	−27	−76	−33	−26	−5	−21
22	−110	−106	+37	−60	+25	−46	−64	−28	−72	−31	−15	−39
23	−70	−61	+34	−32	+126	+35	−59	+34	−69	−38	−31	−46
24	+21	−15	+37	+14	+188	−5	−57	+42	−65	−32	−40	−46

* See § 8.

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TABLE IB.—Lunar Diurnal Magnetic Variations at Pavlovsk Determined from the Mean of a Number of Whole Lunations.*

Twenty-five hourly values are given, separated by intervals of one solar hour, there being twenty-five solar hours in a lunar day.

The unit is 10^{-7} C.G.S. for force, and $0'\!\cdot\!001$ for declination; + denotes force increasing to North, West, and the Nadir.

(Values below are given as signed numbers; + and − correspond to the two sign-columns of the original table.)

Lunar hour	Horizontal force Summer	Horizontal force Equinox	Horizontal force Winter	Horizontal force Year	Declination Summer	Declination Equinox	Declination Winter	Declination Year	Vertical force Summer	Vertical force Equinox	Vertical force Winter	Vertical force Year
0	−28	−30	+21	−26	+320	+117	−29	+136	−6	−4	+1	−4
1	+80	+22	−21	+41	+344	+56	−37	+121	+9	−27	−3	−7
2	+112	+29	−3	+46	+188	−27	−36	+42	+10	−18	−3	−2
3	+134	+34	−25	+48	+66	−108	+33	−3	+9	−9	−11	−4
4	+35	+31	−35	+10	−122	−120	+56	−62	−12	−24	−21	−19
5	+34	+32	−24	+14	−204	+10	+78	−39	−18	−0	−11	−10
6	−9	+17	−1	+2	−282	−57	+52	−96	−6	+14	−13	−2
7	−49	−0	+11	−13	−273	−19	+42	−84	−6	+27	−3	+6
8	−100	−16	+13	−34	−106	−76	−38	−73	−12	+46	+8	+14
9	−78	−42	+31	−30	−17	−6	−22	−15	−32	+33	−3	−1
10	−69	−30	−39	−46	+93	+71	−47	+39	−23	+27	−15	−4
11	−49	−10	−27	−29	+214	+103	−67	+83	−3	+25	−16	+2
12	−11	−14	−36	−7	+214	+119	−67	+89	+22	−1	+2	+7
13	+59	+25	−24	+32	+232	+103	−87	+83	+20	−8	+2	+5
14	+79	+61	−33	+31	+144	+92	+0	+79	+26	−13	+5	+6
15	+93	+48	−16	+37	+70	+43	+86	+66	+21	−9	+24	+12
16	+120	+34	+14	+62	−162	+93	+145	+25	+22	−14	+25	+11
17	+79	+52	+38	+52	−227	−48	+60	−72	+16	−14	+22	+8
18	+15	+38	+38	+13	−262	−78	+56	−95	+3	+1	+23	+9
19	−16	−59	+38	−31	−234	−93	+7	−107	+2	−6	+12	+3
20	−77	−66	+43	−54	−285	−32	+63	−131	+3	−10	−4	−4
21	−94	−63	−1	−48	−163	−89	−77	−63	−2	−6	−3	−4
22	−96	−67	+16	−39	+16	−35	−0	−6	−5	−4	−1	−3
23	−80	−57	+31	−65	+159	−40	−72	+16	−9	−2	−2	−4
24	−119	+39	−18	+20	+290	+31	−109	+71	−29	+2	−11	−13

* See § 8.

TABLE IIA.—Fourier Coefficients of the Lunar Diurnal Magnetic Variations, Reduced to the Epoch of New Moon.

Pola.   Horizontal Force.

The unit of force is 10^{-7} C.G.S.; the small corrections mentioned in §§ 5, 6 have not been applied at this stage, cf. § 9 ; + denotes increasing horizontal force.

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer . . .	1	14	91	85	132	78	69	22	11
	2	− 3	150	62	103	86	67	29	36
	3	119	31	44	119	35	64	19	13
	4	− 3	109	59	139	83	71	3	4
	5	56	74	167	95	87	41	54	12
	6	8	178	107	193	95	80	− 11	20
	7	109	10	108	118	65	61	4	− 9
	8	17	110	43	83	24	43	− 24	8
	Mean . .	40	94	84	123	69	62	12	12
Equinox . . .	1	− 59	26	84	114	50	42	19	− 16
	2	13	92	16	85	53	45	25	38
	3	26	80	59	72	36	25	28	− 13
	4	50	16	21	66	42	72	14	− 4
	5	28	26	60	50	43	69	14	− 4
	6	81	− 5	16	60	39	48	33	29
	7	3	102	71	65	95	51	9	28
	8	− 36	94	51	76	38	66	7	23
	Mean . .	13	54	47	73	50	52	19	10
Winter . . .	1	50	− 40	− 21	− 14	− 16	− 32	− 14	− 10
	2	− 97	51	− 34	27	14	16	10	6
	3	− 42	− 54	7	− 10	5	− 6	6	22
	4	− 15	− 57	33	− 17	13	− 10	− 21	− 17
	5	64	− 86	40	6	12	23	− 4	− 19
	6	2	− 41	23	− 23	− 18	16	38	11
	7	82	− 83	52	− 69	− 2	− 26	3	12
	8	139	− 53	52	3	9	− 43	− 9	− 16
	Mean . .	23	− 45	19	− 12	2	− 8	1	− 1

TABLE IIB.—Fourier Coefficients of the Lunar Diurnal Magnetic Variations, Reduced to the Epoch of New Moon.

Pavlovsk.   Horizontal Force.

The unit of force is 10^{-7} C.G.S.; the small corrections mentioned in §§ 5, 6 have not been applied at this stage, *cf.* § 9; + denotes increasing horizontal force.

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer . . .	1	$-$ 3	90	$-$ 2	103	$-$ 8	61	$-$ 22	9
	2	19	107	12	104	$-$ 14	82	$-$ 22	$-$ 2
	3	$-$ 30	6	$-$ 25	80	$-$ 39	23	$-$ 19	1
	4	62	55	23	77	$-$ 13	79	$-$ 22	7
	5	43	26	36	141	10	75	$-$ 35	$-$ 2
	6	29	106	$-$ 6	167	27	69	$-$ 69	0
	7	53	2	30	71	21	46	5	7
	8	4	51	22	88	$-$ 10	50	0	23
	Mean . .	22	55	11	104	$-$ 3	61	-- 23	5
Equinox . . .	1	$-$ 102	$-$ 2	40	76	$-$ 5	60	23	13
	2	$-$ 55	66	$-$ 41	11	18	61	$-$ 34	15
	3	90	84	11	66	13	5	6	9
	4	42	$-$ 1	$-$ 1	73	$-$ 33	79	$-$ 22	26
	5	$-$ 49	46	$-$ 17	$-$ 8	$-$ 10	77	$-$ 10	34
	6	15	43	$-$ 48	5	1	46	$-$ 5	56
	7	10	38	36	78	50	99	$-$ 29	27
	8	$-$ 27	48	26	86	$-$ 5	68	1	7
	Mean . .	$-$ 10	40	1	48	4	62	$-$ 9	23
Winter . . .	1	45	$-$ 39	$-$ 29	$-$ 32	$-$ 1	$-$ 28	13	$-$ 1
	2	$-$ 77	43	$-$ 49	4	$-$ 16	10	$-$ 1	$-$ 11
	3	$-$ 57	0	$-$ 54	8	$-$ 30	-- 7	12	$-$ 3
	4	$-$ 18	$-$ 55	$-$ 23	0	$-$ 34	$-$ 6	12	2
	5	9	$-$ 30	8	$-$ 20	$-$ 40	$-$ 2	$-$ 34	7
	6	$-$ 35	$-$ 23	3	-- 17	$-$ 42	1	$-$ 18	13
	7	49	$-$ 27	$-$ 16	$-$ 31	10	$-$ 12	$-$ 11	20
	8	90	1	11	28	6	$-$ 17	29	$-$ 9
	Mean . .	1	$-$ 16	$-$ 19	$-$ 8	$-$ 18	$-$ 8	0	2

TABLE IIIA.—Fourier Coefficients of the Lunar Diurnal Magnetic Variations,
Reduced to the Epoch of New Moon.

Pola.   Declination West.

The unit is $0'\cdot001$; the small corrections mentioned in §§ 5, 6 have not been applied at this stage, cf. § 9; + denotes increasing transverse force.

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer	1	206	− 35	235	27	124	− 25	− 42	− 21
	2	153	− 134	249	44	85	40	− 16	14
	3	199	− 50	201	74	103	30	− 16	− 17
	4	62	− 174	147	17	123	− 68	− 17	− 20
	5	46	− 89	270	− 67	114	− 39	14	− 6
	6	192	− 206	185	4	92	8	− 12	26
	7	69	− 135	259	16	109	− 20	27	17
	8	131	− 158	263	20	91	− 4	− 10	− 26
	Mean . .	132	− 123	226	17	105	− 10	− 9	− 4
Equinox	1	108	− 42	58	− 18	37	− 55	49	− 46
	2	46	− 109	40	− 6	69	25	44	− 12
	3	74	− 72	21	15	7	− 57	− 26	− 59
	4	64	− 47	11	82	47	5	22	13
	5	161	− 47	52	112	72	− 16	12	0
	6	114	− 40	89	56	131	5	60	5
	7	25	− 98	180	− 16	38	− 121	88	− 19
	8	91	− 164	119	6	80	− 13	− 6	10
	Mean . .	85	− 77	71	27	60	− 28	30	− 14
Winter	1	− 48	− 43	− 108	77	− 1	54	− 61	51
	2	− 136	− 52	− 14	27	− 25	36	− 9	− 5
	3	− 46	74	− 42	169	7	11	39	11
	4	− 32	− 112	− 13	31	− 22	2	18	7
	5	− 64	− 52	− 58	29	− 1	10	− 16	− 20
	6	− 75	− 63	− 52	88	− 29	17	17	21
	7	− 98	− 38	− 9	25	− 56	66	− 14	− 31
	8	89	− 60	− 94	20	5	− 38	19	− 3
	Mean . .	− 51	− 43	− 49	58	− 15	20	− 1	4

TABLE IIIB.—Fourier Coefficients of the Lunar Diurnal Magnetic Variations,
Reduced to the Epoch of New Moon.

Pavlovsk.   Declination West.

The unit is $0'\cdot001$; the small corrections mentioned in §§ 5, 6 have not been applied
at this stage, *cf.* § 9 ; + denotes increasing transverse force.

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer . . .	1	339	20	249	− 16	95	84	− 18	24
	2	308	− 221	209	− 103	47	19	26	11
	3	333	− 96	180	86	− 20	54	− 11	− 14
	4	157	− 106	95	116	73	− 26	70	5
	5	117	− 53	446	− 9	52	− 13	− 9	21
	6	309	− 237	291	− 61	150	− 57	− 11	− 3
	7	174	− 209	268	16	− 10	− 53	16	− 54
	8	180	− 23	395	− 52	75	24	− 36	15
	Mean . .	240	− 116	267	− 3	58	5	3	1
Equinox . . .	1	183	− 150	84	− 3	18	− 23	82	− 13
	2	201	42	4	− 166	76	31	31	24
	3	127	− 163	− 78	2	2	4	22	− 98
	4	206	7	− 91	117	60	− 43	− 9	9
	5	193	− 39	126	115	− 37	65	100	− 13
	6	370	66	54	34	88	− 13	44	− 20
	7	− 203	− 137	283	− 66	− 2	− 89	− 78	− 25
	8	49	− 268	175	11	6	33	4	31
	Mean . .	141	− 80	70	5	26	− 4	25	− 13
Winter . . .	1	− 5	− 21	− 60	121	− 35	44	2	− 107
	2	− 85	14	− 22	− 17	− 91	32	29	− 58
	3	8	32	− 134	144	39	− 14	17	− 24
	4	− 61	− 113	13	70	− 34	69	37	10
	5	− 17	18	− 82	35	22	49	− 10	− 35
	6	− 33	− 93	− 17	33	− 91	7	1	40
	7	− 43	− 139	5	− 7	− 107	26	69	− 38
	8	*(183)	− 76	− 143	13	24	− 16	12	62
	Mean . .	− 34	− 47	− 55	49	− 34	25	20	− 19

* This value has been rejected in taking the mean.

TABLE IVA.—Fourier Coefficients of the Lunar Diurnal Magnetic Variations, Reduced to the Epoch of New Moon.

Pola.　Vertical Force.

The unit of force is 10^{-7} C.G.S.; the small corrections mentioned in §§ 5, 6 have not been applied at this stage, *cf.* § 9; + means increasing downward force.

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer . . .	1	− 49	49	− 44	35	− 18	31	− 2	9
	2	12	− 17	− 48	36	− 21	10	9	− 4
	3	11	9	− 46	36	− 43	16	− 10	− 10
	4	− 54	16	− 51	47	− 24	33	− 0	6
	5	4	11	− 42	48	− 8	22	− 6	0
	6	− 18	28	− 47	71	− 14	22	− 9	0
	7	23	− 21	− 50	33	− 26	31	− 0	10
	8	− 26	− 2	− 48	11	− 21	− 2	− 17	11
	Mean . .	− 12	9	− 47	40	− 22	20	− 4	3
Equinox . . .	1	− 52	50	17	25	13	9	5	7
	2	− 28	− 27	− 32	7	− 29	10	5	6
	3	− 71	56	− 21	14	6	5	7	15
	4	52	− 2	− 13	54	− 3	21	− 10	19
	5	− 1	− 57	− 15	5	− 25	10	− 15	14
	6	− 69	31	7	56	− 3	− 9	− 27	26
	7	38	− 20	32	20	27	45	22	11
	8	15	− 19	− 25	5	− 8	17	− 1	13
	Mean . .	− 14	2	− 6	23	− 3	13	− 2	14
Winter . . .	1	− 30	− 23	− 7	− 10	− 10	− 8	− 13	− 10
	2	53	48	− 34	40	3	7	15	10
	3	− 68	2	− 55	19	− 29	1	2	− 21
	4	− 10	51	− 46	27	− 1	− 13	− 15	− 2
	5	− 18	− 54	− 24	31	1	3	6	− 2
	6	− 4	31	− 18	7	− 6	− 5	8	1
	7	− 0	19	− 25	25	− 5	0	3	5
	8	− 21	− 13	− 27	− 7	− 5	12	− 1	19
	Mean . .	− 12	8	− 30	17	− 7	0	1	0

TABLE IVb.—Fourier Coefficients of the Lunar Diurnal Magnetic Variations,
Reduced to the Epoch of New Moon.

Pavlovsk.   Vertical Force.

The unit of force is 10^{-7} C.G.S.; the small corrections mentioned in §§ 5, 6 have not
been applied at this stage, *cf.* § 9 ; + means increasing downward force.

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer	1	39	38	− 15	31	11	4	3	− 10
	2	− 76	28	4	29	16	− 2	− 19	8
	3	− 34	− 20	6	− 2	4	5	− 4	− 9
	4	− 70	24	10	30	10	− 4	9	7
	5	55	88	− 1	19	37	− 9	3	− 9
	6	7	− 14	34	− 4	13	− 1	− 1	9
	7	− 10	17	13	− 5	5	− 8	− 5	− 15
	8	− 63	78	− 6	− 3	− 15	− 6	9	10
	Mean . .	− 19	30	6	12	10	− 3	− 1	− 1
Equinox	1	10	34	− 9	− 30	8	− 10	10	8
	2	58	− 20	31	− 19	1	1	− 9	4
	3	− 53	− 47	− 28	− 17	− 13	17	4	7
	4	− 26	55	− 8	13	9	3	− 12	5
	5	− 9	− 13	− 16	− 30	15	0	7	− 11
	6	82	− 67	3	− 5	− 2	− 28	15	3
	7	− 54	− 46	− 10	− 34	− 22	− 6	13	8
	8	− 84	35	− 3	− 8	− 1	12	1	− 1
	Mean . .	− 10	− 9	− 5	− 16	− 1	− 1	4	3
Winter	1	33	34	− 4	11	− 6	− 8	− 14	0
	2	19	29	30	39	4	17	3	− 1
	3	12	− 60	− 26	− 1	17	12	− 7	6
	4	− 57	14	− 8	41	− 12	10	3	− 1
	5	85	30	4	0	− 1	0	− 6	3
	6	38	− 40	− 6	− 18	4	3	− 6	− 3
	7	29	− 38	− 10	− 1	21	− 25	1	− 14
	8	49	− 28	3	− 28	− 10	− 11	6	− 9
	Mean . .	26	− 7	− 2	5	2	0	− 3	− 2

TABLE V.—Probable Errors of the Mean Values of a_n and b_n in Tables II.–IV.*

The results are in every case reduced to force, the unit being 10^{-7} C.G.S.

Observatory and element.	Season.	a_1.	b_1.	a_2.	b_2.	a_3.	b_3.	a_4.	b_4.
Pola. Horizontal force	Summer	13	14	10	8	7	3	6	3
	Equinox	10	12	7	4	4	4	2	6
	Winter	19	8	8	6	3	6	4	4
Pavlovsk. Horizontal force	Summer	8	11	5	8	5	5	5	2
	Equinox	16	7	9	11	6	6	5	4
	Winter	15	7	6	6	6	3	5	2
Pola. Transverse force	Summer	11	10	7	5	2	6	3	4
	Equinox	7	7	9	8	6	8	6	4
	Winter	9	6	6	8	4	5	5	4
Pavlovsk. Transverse force	Summer	13	12	13	9	6	6	4	3
	Equinox	15	15	15	10	5	6	6	4
	Winter	8	9	5	7	7	4	3	6
Pola. Vertical force	Summer	8	5	1	4	2	3	2	2
	Equinox	13	10	6	5	4	3	4	1
	Winter	7	10	4	5	2	2	2	3
Pavlovsk. Vertical force	Summer	13	9	3	5	3	1	2	3
	Equinox	14	12	4	4	3	3	2	1
	Winter	8	11	3	6	3	3	2	1

* *Cf.* § 10.

TABLE VIA.—Fourier Coefficients of the Lunar Diurnal Variations of the Geographical Components of Magnetic Force at Pavlovsk and Pola,* in the Formula

$$\Sigma C_n \sin (nt + \theta_n).$$

The time $t = 0$ is the local time of upper culmination of the moon at the phase of new moon. The unit of force is 10^{-7} C.G.S. ; + denotes an increase of force to the North, West, or to the Nadir.

n.	North force.				West force.				Vertical force.			
	Pavlovsk.		Pola.		Pavlovsk.		Pola.		Pavlovsk.		Pola.	
	C_n.	θ_n.	C_n.	θ_n.	C_n.	θ_n.	C_n.	θ_n.	C_n.	θ_n.	C_n.	θ_n.
Summer.												
1	61	23·8	111	14·5	131	117·9	113	125·8	36	329·5	15	308·1
2	105	8·3	133	27·2	128	92·7	161	79·9	13	27·1	62	311·4
3	62	− 0·9	86	43·3	28	87·3	80	88·2	11	106·6	30	313·3
4	26	295·2	20	47·5	2	—	− 4†	(91)	1	—	5₅	(308)
Equinox.												
1	42	− 11·2	63	4·6	79	120·7	71	126·2	13	230·5	14	278·7
2	48	3·1	80	31·0	82	88·0	60	62·1	17	199·0	24	346·5
3	63	5·4	71	39·3	13	100·3	48	102·3	2	—	14	348·1
4	27	− 18·2	21	54·9	15	119·6	26	109·2	− 5†	(233)	14	352·8
Winter.												
1	17	176·6	50	145·8	28	218·0	46	220·5	28	107·9	15	304·8
2	20	250·2	30	127·8	36	313·8	45	322·2	6	341·0	35	300·4
3	20	249·6	11	161·2	21	308·4	15	321·7	2	—	7	270·8
4	2	—	2	—	− 15†	315·6	3	(351)	4	(231)	1	—

* *Cf.* § 11.

† The coefficients C_n of these components are reversed in sign in order to exhibit the accordance between the values of θ_n for different values of n.

TABLE VIb.—Fourier Coefficients of the Lunar Diurnal Variations of the Geographical Components of Magnetic Force at Pavlovsk and Pola,* in the Formula

$$\Sigma \left(a_n \cos nt + b_n \sin nt\right).$$

The time $t = 0$ is the local time of upper culmination of the moon at the phase of new moon; the unit of force is 10^{-7} C.G.S.; + denotes an increase of force to the North, West, or to the Nadir.

$n.$	North force.				West force.				Vertical force.			
	Pavlovsk.		Pola.		Pavlovsk.		Pola.		Pavlovsk.		Pola.	
	$a_n.$	$b_n.$	$a_n.$	$b_n.$	$a_n.$	$b_n.$	$a_n.$	$b_n.$	$a_n.$	$b_n.$	$a_n.$	$b_n.$
Summer.												
1	25	56	28	107	116	− 61	91	− 66	− 18	31	− 12	9
2	15	104	61	119	128	− 6	158	28	6	12	− 46	41
3	− 1	62	59	62	28	1	78	2	10	− 3	− 22	21
4	− 25	7	14	13	2	1	− 4	0	− 1	− 1	− 4	3
Equinox.												
1	− 8	41	5	63	68	− 40	57	− 42	− 10	− 8_5	− 14	2
2	3	48	41	68	82	3	53	28	− 5_5	− 16	− 6	23
3	6	63	45	55	13	− 2	47	− 10	− 1	− 1	− 3	13
4	− 9	26	17	12	13	− 7	24	− 8	4	3	− 2	14
Winter.												
1	0	− 17	28	− 41	− 17	− 22	− 30	− 35	26	− 8_5	− 12	8
2	− 19	− 7	24	− 18	− 26	25	− 28	35	− 2	5_5	− 30	18
3	− 19	− 7	4	− 10	− 16	13	− 9	12	2	0	− 7	0
4	0	2	1	− 2	11	− 11	0	3	− 3	2_5	1	0

* *Cf.* § 11.